Exact Solution of an Unsteady Boundary-Layer Flow Past a Wedge Considering the Magnetic Field Effects

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Abstract— The present paper is on the study of two-dimensional magnetohydrodynamic unsteady boundary-layer flow of an incompressible laminar viscous fluid over a wedge in which the outer freestream velocity is assumed to be proportional to the distance along the wedge surface. The model is described by the unsteady Falkner-Skan equation (UFSE) that accounts various physical parameters. Using time dependent similarity transformations the governing equations are transformed in to ordinary differential equation, and we explore the effects of magnetic fields on the flow analytically. In order to compare our exact solutions, the UFSE is also solved numerically. Obtained results show that effects of accelerated pressure gradient and magnetic parameter are to increase the velocity of the fluid and the momentum boundary-layer thickness and whereas the decelerated pressure gradient promotes the oscillatory type solutions. We note that suction and magnetic field suppress the oscillations of the velocity profiles. For unsteady parameter k, when we have decelerated flow we see initially the boundary-layer thickness increases and then it decreases. The effects of all the involved parameters on velocity profiles, skin friction coefficient are discussed and their results are interpreted with the aid of the graphs.

Keywords— Magnetic Field, Pressure Gradient, Suction /injection, Unsteady Boundary-layer.

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1 Introduction

MHD is considered as one of the richest fields which describe the motion of the electrically conducting media in the presence of magnetic field which has stabilizing effects on the boundary-layer flow. From the past decades, we have a active research in this field, because MHD boundary-layer has many applications in the field of metallurgical processes, technical processes, engineering processes such as power generators, the cooling reactors, polymer industry, spinning of filaments, flow meters and pumps, the design of heat exchangers and accelerators, plasma studies, geothermal energy extraction, oil exploration, the cooling of electronic devices and nuclear reactors during emergency shutdown is encountered. The study of MHD flow was started in the approximation of the boundary-layer theory by Pavlov [1] where the problems of the fluid flow was solved for an electrically conducting fluid in presence of transverse magnetic field. In this direction we have numerous papers on MHD boundary-layer like Rossow [2] has given a report stating that the motion of the electrically conducting fluid can be controlled using magnetic field, the combined effect of viscoelasticity and magnetic field was considered by Anderrson [3], the Hall effects on classical MHD boundary-layer flow was given by Watanabe and Pop [4].

Suction/injection has important role in the field of aerodynamics, space sciences, etc., and the chief reason why boundary-layer suction is of interest are that it causes separation to be delayed, and that it exerts a stabilizing influence on laminar flow. Suction opposes an adverse pressure gradient and makes the velocity profiles more concave, this leads to reduction in the boundary-layer thickness. The effects of blowing are less interesting since they oppose the stability of the boundary-layer and the attachment of the layer to the surface. The boundary-layer is thickened and blown out from the surface. It is well-known that the effects of injection on the boundary-layer flow are of interest in reducing the drag. It also finds applications in polymer technology, metallurgy, and dyeing industries. The studies for unsteady flow with effects of suction/injection was given by Attia [5], Uwanta and Hamza [6], with porous plate by Jha et al. [7], MHD flow over permeable stretching surface including suction/injection was made by Das et al. [8], Pop and Na [9] have shown that the suction reduces the boundary-layer thickness whereas injection has the opposite effects in the presences of magnetic field and thermal radiation, the transition effects of the MHD boundary-layer flow was studied by Kumaran et al. [10], Takhar et al. [11], Turkyilmazoglu and Pop [12], Nandkeolyar et al. [13] etc. Numerical solution of MHD flow in the presence of magnetic field with and without suction/injection has been given Cobble [14], Soundalgekar [15]. The control of the fluid on the surface of subsonic aircraft using suction/injection was given by Shojaefard et al. [16]. MHD problem related to unsteady arise in a wide variety like form the prediction of space weather, origin of earth's magnetic field, damping of turbulent fluctuations in semiconductors, measurements of the flow rates of beverages in food industry. Inviscid theory is used to study the overall performances of electromagnetic pumps and generators and boundary-layer theory is used to study the wall shear stress. Along with these plenty of research paper there are books on MHD like Pai [17], Shereliff [18], Sutton and Sherman [19], Hughes and Young [20], etc.

As pointed out by Hasting and Troy [21] for classical Falkner-Skan equation, solutions in the current problem for MHD Falkner-Skan flow also oscillate a finite number of times, but eventually tend to the prescribed boundary condition. This motivates us to find the exact solution of unsteady boundary-layer with effects of magnetic fields and found that for increasing magnetic field it is true. Nevertheless, according to the authors knowledge there is no exact solution reported on

MHD unsteady boundary-layer with all parameters involved in our analysis, therefore an attempt is made, to find an exact solution to the two-dimensional MHD unsteady boundary-layer equation for general values of pressure gradient (β), magnetic parameter (Hartman number) (M), suction/injection parameter (α) and stretching parameter (λ).

2 Formulation of the Problem

A viscous and incompressible fluid flow over a moving wedge with a velocity Uw(x,t) in the opposite direction to the outer free stream velocity U(x,t) and the moving wedge is permeable with lateral mass flux velocity Vw(x,t) is considered for MHD two-dimensional unsteady laminar boundary-layer. The flow is in the upper half-space y>0, if x measures distance from the leading edge of wedge surface and y-direction is normal to it. For large Reynolds number the viscosity effects are confined to a thin layer near the wedge surface due this we have near field where the role of viscosity is important and role of zero-shear viscosity is important in far field. The moving wedge is subjected to applied magnetic field.

Governing equations for unsteady MHD boundary-layer flow over wedge are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

where u and v are the velocity components in the x and y-direction, υ is the kinematic viscosity of the fluid, σ is the electrical conductivity, ρ is the fluid density and the magnetic field is given by B(x). From (3) we mean that the pressure in the boundary-layer is same as the pressure in the mainstream flow U(x,t) (from the influences of the applied magnetic field, the pressure is constant in the inviscid flow (i.e the outer velocity u=U(x,t), v=0)) as

$$-\frac{1}{\rho}\frac{dp}{dx}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}-\frac{\sigma B^2}{\rho}U.$$

Accordingly the pressure gradient in (2) becomes

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\partial U}{\partial t}-\frac{\sigma B^2(x)}{\rho}(u-U) \tag{4}$$

The relevant boundary conditions are

$$y=0:\quad u=-U_w(x,t),\quad v=-V_w(x,t), \text{ and}$$
$$y\to\infty:\ u\to U(x,t) \tag{5}$$

where $U(x,t)$ is the velocity at the edge of the boundary-layer, $U_w(x,t)$ is the stretching surface velocity which obey the power law relations

$$U(x,t)=U_\infty A(t)x^m,\ U_w(x,t)=U_{\infty w}A(t)x^m,\ A(t)>0 \tag{6}$$

U_∞, $U_{\infty w}$ and m are constants and A(t) is a general function of time. And also $B(x)=B_0 x^{(m-1)/2}$, where B_0 the constant magnetic field, x is is the distance measure from the outset of the boundary-layer. Introducing the stream function $\psi(x,y,t)$ as

$$(u,v)=\left(\frac{\partial\psi}{\partial y},-\frac{\partial\psi}{\partial x}\right) \tag{7}$$

which satisfies the continuity equation identically, and considering the similarity transformations,

$$\psi(x,y,t)=\sqrt{\frac{2\upsilon xU}{1+m}}\,f(\eta),\quad \eta=\sqrt{\frac{(1+m)U}{2\upsilon x}}\,y. \tag{8}$$

Plugging (6), (7) and (8) into (4), we get MHD unsteady Falkner-Skan equation as

$$f'''(\eta)+f(\eta)f''(\eta)+\beta\left(1-f'^2(\eta)\right)-M^2\left(f'(\eta)-1\right)=k\left(\frac{\eta}{2}f''(\eta)+f'(\eta)-1\right) \tag{9}$$

with the boundary conditions

$$f(0)=\alpha,\qquad f'(0)=-\lambda\qquad f'(+\infty)=1, \tag{10}$$

where $\beta=\frac{2m}{1+m}$ is the pressure gradient parameter with $\beta>0\,(<0)$ represents the accelerating (decelerating) flow, $M=\sqrt{\frac{2\sigma}{\rho A(t)U_\infty(1+m)}}B_0$ is magnetic parameter,

$$k=\left(\frac{2}{m+1}\frac{U_\infty^{(m-2)}}{\upsilon^{m-1}}\right)\frac{A'(t)}{A^2(t)}, \tag{11}$$

is a dimensionless measure of the unsteadiness of the flow, $\alpha=\sqrt{\frac{2\rho x}{(1+m)\mu U(x,t)}}V_w(x,t)$ is the normal relative velocity, $\alpha>0$ represents suction and $\alpha<0$ represents injection, whereas $\alpha=0$ is impermeable of the plate, and $\lambda=-\frac{U_{\infty w}}{U\infty}$, the boundary has a specified velocity and stretch. For $\lambda>0$, the wedge is moving opposite to the mainstream flow. Integrating (9) and (10) with $\beta=-1, k=0, M=0$ gives us Riccati type equation

$$f'(\eta)+\frac{f^2(\eta)}{2}=\frac{\eta^2}{2}+\Delta\eta-\alpha\lambda\eta+\frac{\alpha^2}{2}-\lambda \tag{12a}$$

where $\Delta = f''(0)$, which has an exact solution

$$f(\eta)=\eta+(\Delta-\alpha\lambda)+\frac{(\alpha(1+\lambda)-\Delta)e^{-(\frac{\eta^2}{2}+\eta(\Delta-\alpha\lambda))}}{1+(\alpha(1+\lambda)-\Delta)\sqrt{\frac{\pi}{8}}\,e^{\frac{(\Delta-\alpha\lambda)^2}{2}}\left(erf\left(\frac{\eta+\Delta-\alpha\lambda}{\sqrt{2}}\right)\right)-erf\left(\frac{\Delta-\alpha\lambda}{\sqrt{2}}\right)} \quad (12b)$$

provided $\Delta=\alpha\lambda\pm\sqrt{-2(1+\lambda)+\alpha^2}$, the velocity profiles for different values of α and λ can be obtained by differentiating (12b). The velocity gradient at the wall will be $f''(0)=\Delta=\alpha\lambda\pm\sqrt{-2(1+\lambda)+\alpha^2}$. The exact solution of the (9) and (10) is $f(\eta)=\eta+\alpha$ for $\lambda=-1,\ \alpha=0$ then the velocity gradient becomes zero, which demarcate the solution nature across boundary-layer flow.

3 EXACT SOLUTION

The aim of the paper is to obtain the exact solution (9) and (10) for all λ,β,M,α and k we shall now rewrite the (12b) in the form

$$f(\eta)=\eta+(\Delta-\alpha\lambda)-\frac{\Delta-(\alpha(1+\lambda))}{G(\eta)} \quad (13)$$

where

$$G(\eta)=\left(e^{-\frac{(\Delta-\alpha\lambda)^2}{2}}-\sqrt{\frac{\pi}{2}}\left(\frac{\alpha(1+\lambda)-\Delta}{2}\right)erf\left(\frac{(\Delta-\alpha\lambda)}{\sqrt{2}}\right)\right)e^{\frac{(\eta+(\Delta-\alpha\lambda))^2}{2}}$$

$$+\sqrt{\frac{\pi}{2}}\left(\frac{\alpha(1+\lambda)-\Delta}{2}\right)e^{\frac{(\eta+(\Delta-\alpha\lambda))^2}{2}}erf\left(\frac{\eta+(\Delta-\alpha\lambda)}{\sqrt{2}}\right) \quad (14)$$

the (14) is obtained on adding and subtracting $\Delta^2/2$ to the exponent in the numerator of (12b) and simplifying it, we obtain $G(\eta)$ in the above form. Substituting (13) into UFSE (9) and (10) we get

$$G^2G'''-GG''(6G'-(\alpha(1+\lambda)-\Delta)-\frac{k}{2}\eta)G)-G^2G'(2\beta+M^2+k)$$

$$-G'^2((\alpha(1+\lambda)-\Delta)(2-\beta)+((2-k)\eta-2(\Delta-\alpha\lambda))G)+6G'^3=0 \quad (15)$$

where $G=G(\eta)$, and corresponding boundary conditions become

$$G(0)=1,\ G'(0)=\frac{(1+\lambda)}{\alpha(1+\lambda)-\Delta},\ G'(+\infty)=0 \quad (16)$$

the solution of (15) and (16) is given by (14). The error and exponential functions in (14) may be expanded in terms of Taylor series expansion which have infinite radius of convergence. This solution (14) for $\beta=-1,\ k=0$ and $M=0$ gives a hint to obtain a similar analysis for the other values of $\beta,\ M$ and k. It is therefore natural to assume the base solution as

$$G(\eta)=\sum_{n=0}^{\infty}a_n\eta^n \quad (17)$$

where the coefficients a_n are to be determined as a function of β,k,M and Δ. The convergence of the above series is postponed for a while. The first two coefficients $a_0=1,\ a_1=\frac{(1+\lambda)}{\alpha(1+\lambda)-\Delta}$ are known from the boundary conditions (16)

$$a_3=\frac{-1}{6(\alpha-\Delta+\alpha\lambda)^3}(1+\lambda)(6-k\Delta^2+M^2\Delta^2-\beta\Delta^2+12\lambda+\beta\Delta^2\lambda+6\lambda^2$$

$$+2\alpha\Delta(1+\lambda)(1+k-M^2+\beta+\beta\lambda))$$

$$+2(\alpha-\Delta+\alpha\lambda)^2(-6(1+\lambda)+\alpha(\alpha-\Delta+\alpha\lambda))\alpha_2$$

etc., and substitution of (17) in (15) gives the recurrence relation (which involves tedious algebra)

$$a_{n+3}=\frac{-1}{(n+1)(n+2)(n+3)}$$

$$(\sum_{j=0}^{n-1}\sum_{i=0}^{n-j}(j+1)(j+2)(j+3)a_ia_{n-j-i}a_{j+3}$$

$$-\sum_{j=0}^{n}((\alpha(1+\lambda)-\Delta)(j+1)(j+2)a_{n-j}a_{j+2}$$

$$-(2-\beta)(n-j+1)a_{j+1}a_{n-j+1})$$

$$-\sum_{j=0}^{n}(i+1)(j-i+1)(2(\Delta-\alpha\lambda)a_{n-j}$$

$$-6(n-j+1)a_{n-j+1})a_{i+1})a_{j-i+1}$$

$$+\sum_{i=0}^{n-j}((j+1)(ja_{j+1}+(\Delta-\alpha\lambda)(j+2)a_{j+2}$$

$$-\left(2\beta+M^2+k(j+2)/2\right)a_{j+1}a_ia_{n-j+1}$$

$$-(i+1)(6(j+1)(j+2)a_{j+2}$$

$$+(2+k)(n-j-i)a_j)a_{i+1}a_{n-j-i}))) \quad (18)$$

for $n=1,2,3......$.

From (18) we can notice that all the coefficient a_n have been obtained in terms of β,k,M,α. The above coefficients a_n involve one free coefficient a_2 which remains unknown due to the derivative boundary condition at infinity, this a_2 characterizes the coefficient of skin friction can be found by (13) and (17) in such way that the derivative condition at far distance is satisfied. This is equivalent of finding value of a_2 of series (17) or $f''(0)$ from the system of (9) and (10) as (13) intrinsically relates this with each other

$$a_2 = \frac{-1}{(2\alpha(1+\lambda)-\Delta)}\left(f''(0) - \frac{2(1+\lambda)^2}{(\alpha(1+\lambda)-\Delta)}\right) \qquad (19)$$

from this either we can obtain a_2 or $f''(0)$ to satisfy the end condition of (10) or (16). To compute a_2 or $f''(0)$ integrate (9), we get

$$f''(0) - \alpha(1+\lambda) = \frac{k}{2}\eta_\infty + \int_0^\infty \left(f' - \frac{k}{2} f' - f'^2 \right) d\eta$$
$$+ \beta \int_0^\infty \left(1 - f'^2\right) d\eta + M^2 \int_0^\infty (f' - 1) d\eta \qquad (20)$$

where η_∞ denotes the large value of η, we now write (20) in more convincing manner and also useful for its integration as

$$f''(0) - \alpha(1+\lambda) = \frac{k}{2}\eta_\infty + \int_0^{\eta_\infty} \left(f' - \frac{k}{2} f' - f'^2 \right) d\eta$$
$$+ \beta \int_0^{\eta_\infty} \left(1 - f'^2\right) d\eta + M^2 \int_0^{\eta_\infty} (f' - 1) d\eta \qquad (21)$$

the solution of the above asymptotic integral relation is too complicated by the fact that the boundary condition is specified at infinity. Looking at the equation (19) and (20), $f''(0)$ appears on both sides therefore it should be found iteratively using suitable initial approximation and (21) provides us to choose an initial approximation for $f''(0)$. From the known exact analytical solution (12b) for all parameters β, k, M and α which is good initial estimate for $f''(0)$, and also it ensure the fast convergence. Thus, within a few integrations, we obtain $f''(0)$ which is accurate enough. The derivative condition at far distance is satisfied i.e., $f'(\eta) \to 1$ as $\eta \to \infty$. The values of $f''(0)$ which defines the skin friction coefficient thus obtained are consistent with numerical solution of the problem. The results obtained by the method described above have been presented in terms of velocity profiles in figures, and of the skin friction coefficient in table.

4 RESULTS AND DISCUSSION

In the present paper, the similarity solution of the MHD unsteady Falkner-Skan equation is obtained when the main stream flow is of the form $A(t)x^m$ The dependency of the boundary-layer flow on pressure gradient, unsteady parameter, magnetic parameter (Hartmann number) stretch and speed of the wedge surface parameter and suction/injection parameter have been studied. Following the approach of Sachdev et al. [22] for steady case we have developed a new similar approach for unsteady flow. To validate our obtained results we have given numerical solution of the problem. The obtained results exist for all values of β, k, M and λ which are presented in the form of velocity profiles in Figs. (1- 5) and skin friction co-efficient in table (1).

Fig. 1 is plotted for variation of $f'(\eta)$ with η for an various stretching parameter λ. It is seen that the flow becomes oscillatory for negative pressure gradient and also the velocity curves asymptotically satisfy their end condition, from this it is also clear that the effects of magnetic field is to control oscillation (i.e. overshoot of the velocity profiles are seen) contradiction to Fig. 1 in Fig. 2 which is plotted for different values of decelerated flow parameter $(\beta < 0)$ fixing other parameters, that when there is magnetic effect or suction effects it will suppress the oscillation of the velocity profiles (i.e no overshoot or undershoot of velocity profiles are seen) but surprisingly we see that the curves for $\beta = -2.5, -3.0)$ decrease first i.e., it represents the undershoot $(f'(\eta) < 1$ for some $\eta)$ and eventually satisfy their end condition. This was pointed out by Hastings and Troy [20] for classical Falkner-Skan equation, therefore in the present problem MHD unsteady Falkner-Skan flow also oscillate a finite number of times, but eventually tend to prescribed boundary condition which is true even for increasing magnetic parameter M .

In Fig. 3 the effects of magnetic parameter M on horizontal velocity $f'(\eta)$ shows that the velocity decreases as magnetic parameter (M) increases and the boundary-layer acquires more magnetism that leads to the variation in Lorentz force which opposes the flow, due this the momentum boundary-layer thickness decreases as magnetic parameter increases, from this we can point out that the presence of magnetic field has effects on boundary-layer flow when compared to the results with no magnetic field. The same nature is seen for pressure gradient also in Fig. 4.

It is clear that the effects of pressure gradient and magnetic parameter are to decreases the velocity as results momentum boundary-layer thickness reduces it is observed that all curves no undershoot and overshoot for these parameter. We also draw the velocity shapes $f'(\eta)$ as a function (η) for unsteady parameter k in Fig. 5 for other fixed parameters. It is immediately clear that for increasing k from -3.0 to 3.0, the boundary-layer thickness decreases which means that flow is attached to the moving wedge. In this case profiles are benign and are well within the boundary-layer.

The wall shear stress $f''(0)$ are plotted as a function of k for various values of magnetic parameter in Fig.6 and for various values of pressure gradient in Fig. 7. In Fig.6 we notice that as unsteady parameter increases from zero the wall shear stress gradually decreases to negative infinity. Further these wall shear stress decreases as the magnetic parameter decreases. We can also observe that the velocity profiles exist and satisfy the boundary condition (10). The results of (9) in Fig.7 are plotted for different values of pressure gradient it can be seen that for increasing unsteady parameter a linear decrease in wall shear stress exist and also for each $f''(0)$ the velocity profiles exists.

Table 1 compares the values for the skin friction coefficient $f''(0)$ obtained by exact solution with that of the numerical solution of the problem for various values of β, M, k and α . We see that there is an excellent agreement between two solutions for all values given in the table. Also as the wall

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stretching parameter increases (in absolute sense), skin friction coefficient also increases. Also as β, M and α increases, it again increases.

5 FIGURES AND TABLES

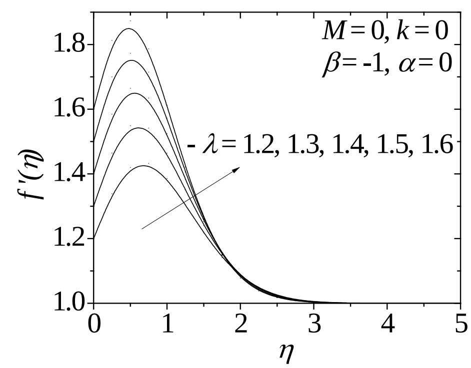

Figure 1: Variation of velocity profiles $f'(\eta)$ with η for different values of λ analytically.

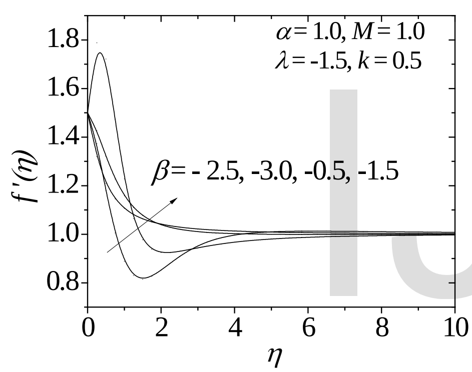

Figure 2: Variation of velocity profiles $f'(\eta)$ with η for different values of pressure gradient $\beta < 0$ analytically.

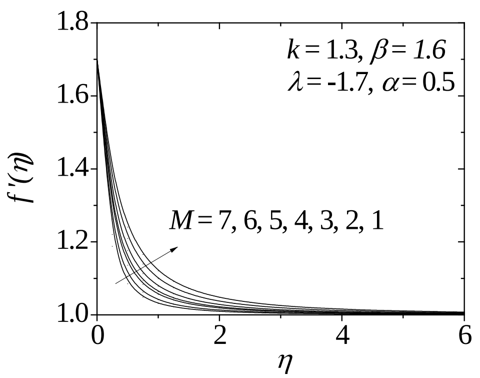

Figure 3: Variation of velocity profiles $f'(\eta)$ with η for different values of magnetic parameter M analytically.

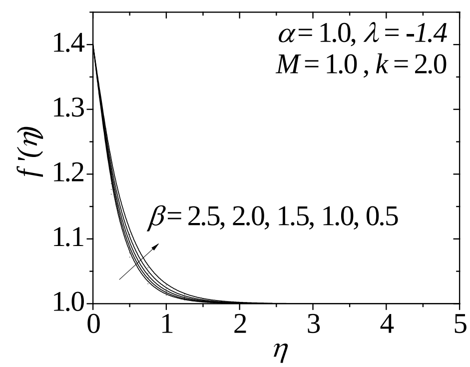

Figure 4: Variation of velocity profiles $f'(\eta)$ with η for different values of Pressure gradient $\beta > 0$ analytically.

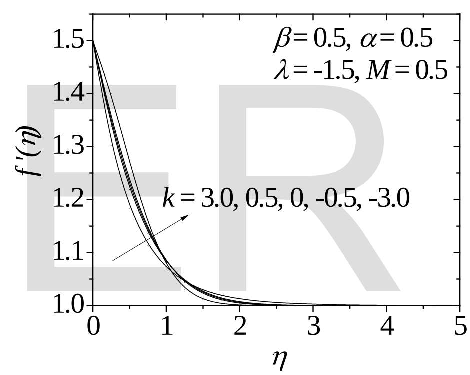

Figure 5: Variation of velocity profiles $f'(\eta)$ with η for different values of unsteady parameter k analytically.

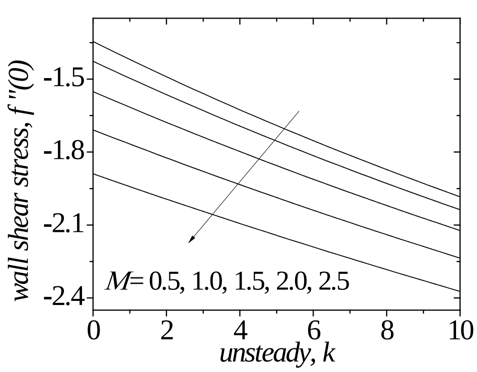

Figure 6: Velocity profiles for the wall shear stress $f''(\eta)$ with unsteady parameter k for different values of magnetic parameter M which are obtained analytically.

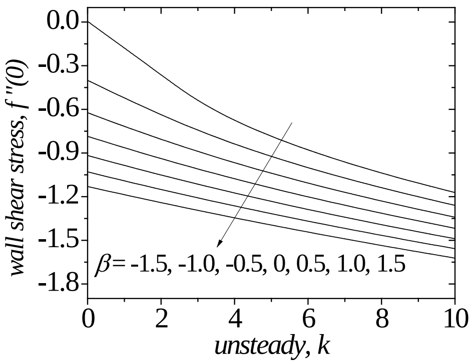

Figure 7: Velocity profiles for the wall shear stress $f''(\eta)$ with unsteady parameter k for different values of pressure gradient β which are obtained analytically.

Table 1: Comparison of wall stress value $f''(0)$ with numerical solution of the problem.

	$k=-1, \beta=1, \lambda=-1.2$					
	M=0		*M*=1		*M*=2	
α	Exact	Num	Exact	Num	Exact	Num
-0.5	-0.2245	--0.2245	-0.2966	-0.2964	-0.4362	-0.4392
-1	-0.1885	-0.1885	-0.2013	-0.2513	-0.3937	-0.3937
0	-0.2882	-0.2882	-0.3489	0.3489	-0.4897	-0.4897
0.5	-0.3498	-0.3498	-0.3977	-0.4077	-0.5452	-0.5452
1	-0.4177	-0.4177	-0.4725	-0.4726	-0.6054	-0.6054
	$k=1, \beta=-1, \lambda=-1.4$					
α	Exact	Num	Exact	Num	Exact	Num
-0.5	0.9407	0.9607	-0.0732	-0.0755	-0.6088	-0.6088
-1	1.5698	1.5698	0.0214	0.02146	-0.5229	-0.5230
0	0.4453	0.4460	-0.2036	-0.2036	-0.6985	-0.7085
0.5	0.0619	0.0620	-0.3547	-0.3547	-0.8216	-0.8216
1	-0.2292	-0.2292	-0.5212	-0.5212	-0.9472	-0.9472

5 CONCLUSIONS

In this paper we have obtained the exact solution of the MHD unsteady Falkner-Skan equation for two-dimensional boundary-layer flow past wedge with suction /injection in the presence of magnetic field. The comparison of the obtained exact method is done numerically and found that it agrees well. It is shown that the effects of pressure gradient and magnetic parameter are to decrease the momentum boundary-layer thickness. As well as the nature of suction ($\alpha > 0$) parameter is different from injection ($\alpha < 0$) parameter i.e., suction always reduces the thickness while injection has an opposite effect. For unsteady parameter k with $k < o$ the boundary-layer thickness increases and then it decreases when k is increased to positive value and for $k > 0$ as k increases the velocity decreases then boundary-layer thickness decreases.

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